Correlation Functions and AdS/LCFT Correspondence

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Abstract

Correlation functions of Logarithmic conformal field theory is investigated using the ADS/CFT correspondence and a novel method based on nilpotent weights and 'super fields'. Adding an specific form of interaction, we introduce a perturbative method to calculate the correlation functions.

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1 Introduction

In this paper, we will examine calculation of correlation functions in a logarithmic conformal field theory, which has been derived using the AdS/CFT correspondence. The correspondence which is by now very popular in the string theory literature, maintains that a d dimensional conformal theory may arise on the boundary of a $d+1$ dimensional theory defined on an Anti de Sitter back ground [1]. Since Logarithmic Conformal Field Theories (LCFTs) are singular forms of CFTs, one expects that they arise on the boundary too, but perhaps as boundary theories of rather unusual bulk theories [2, 3]. The goal of the present discussion is to examine the way one loop correlation functions within LCFT may be calculated, if the bulk theory is given. A method based on Feynman graphs which connect to the boundary, has been suggested by Witten [4], which gives the road map by which correlation functions for the boundary CFT can be calculated. The same method works for LCFTs as well. Except that the number of correlation function to calculate for an LCFT is very high. In the simplest case, where one has a logarithmic pair, there are sixteen interconnected correlation functions when considering four point functions. Recently we proposed a method for dealing with LCFTs based on nilpotent variables [5]. This method offers a unified manner by which one can calculated the correlation functions of LCFTs [6],

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and easily generalizes to the case of AdS/LCFT correspondence [7]. However its application to the graphs with loops was riddled with some difficulties [8]. Here we present a method by which these problems are ironed out and we are able to discuss the properties of loop calculations.

2 AdS/LCFT Correspondence

To state the AdS/LCFT correspondence, we briefly recall the conjecture in the frame work of ordinary CFT. In this conjecture, one relates two different theories: one of them lives in a $d+1$ dimensional AdS space, and the other somehow lives on the boundary of this AdS space and has conformal invariance. Let us be more precise. Consider the action $S[\Phi]$ defined on AdS_{d+1} and calculate the partition function of the AdS theory subjected to the constraint that the value of Φ on the boundary be Φ_b

$$Z_{AdS}[\Phi_b] = \int_{\Phi_b} D\Phi \exp(-S[\Phi]). \tag{1}$$

The correspondence states that the partition function of AdS theory is the generating functional of the boundary conformal field theory:

$$Z_{AdS}[\Phi_b] = \left\langle \exp\left(\int_{\partial AdS} dx \hat{O} \Phi_b \right) \right\rangle. \tag{2}$$

The function Φ_b is considered as a current which couples to the scalar conformal operator $\hat{O}$ via a coupling $\int_{\partial AdS} dx \hat{O}\Phi_b$. This is an elegant and useful result, since it gives a practical way for calculation of correlation functions of conformal field theory. To begin, one should first choose a proper action in the bulk. Different actions have been studied and the conformal correlators have been found in these cases. Some examples are interacting massive scalar field theory [11] or interacting scalar spinor field theory [12].

On the other hand, it is also interesting to find actions on AdS which induce logarithmic conformal field theory (LCFT) on its boundary.

The bulk action which give rise to logarithmic operators on the boundary were first described in [2, 3]. Then it was accustomed to the new method for investigating LCFTs, i.e. via nilpotent and grassmann variables [7]. In this new method a superfield is defined using a grassmannian variable η and different components of a logarithmic pair and fermionic fields

$$\hat{O}(\vec{x}, \eta, \bar{\eta}) = \hat{A}(\vec{x}) + \hat{\bar{\zeta}}(\vec{x})\eta + \bar{\eta}\hat{\zeta}(\vec{x}) + \bar{\eta}\eta\hat{B}(\vec{x}) \tag{3}$$

where $\hat{\zeta}(\vec{x})$ and $\hat{\bar{\zeta}}(\vec{x})$ are fermionic fields with the same conformal dimension as $\hat{A}(\vec{x})$, and $\hat{B}(\vec{x})$ is the logarithmic partner of $\hat{A}(\vec{x})$. Also $\bar{\eta}\eta$ acts as the nilpotent variable. Now it is easy to see that $\hat{O}(\vec{x}, \eta, \bar{\eta})$ has the following transformation law

$$\hat{O}(\lambda\vec{x}, \eta, \bar{\eta}) = \lambda^{-(\Delta + \bar{\eta}\eta)} \hat{O}(\vec{x}, \eta, \bar{\eta}) \tag{4}$$

If $\hat{O}(\vec{x}, \eta, \bar{\eta})$ were the logarithmic operator on the boundary of AdS, the corresponding field $\Phi(x, \eta, \bar{\eta})$ in AdS space can be extended as

$$\Phi(x, \eta, \bar{\eta}) = C(x) + \bar{\eta}\alpha(x) + \bar{\alpha}(x)\eta + \bar{\eta}\eta D(x) \tag{5}$$

where x is $(d+1)$ dimensional coordinate with $x^0, x^1, ..., x^d$ components and $x = (\vec{x}, x^d)$ and $x^d = 0$ corresponds to the boundary. In reference [7] we introduced a free action with BRST symmetry. Now we write it as the following form

$$S^f[\Phi] = -\frac{1}{2}\int_\Omega d^{d+1}x\sqrt{|g(x)|}\int d\bar{\eta}d\eta \times \\ \left[\vec{\nabla}\Phi(x,\eta,\bar{\eta}).\vec{\nabla}\Phi(x,-\eta,-\bar{\eta}) + m^2(\eta,\bar{\eta})\Phi(x,\eta,\bar{\eta})\Phi(x,-\eta,-\bar{\eta})\right] \tag{6}$$

where $m^2(\eta,\bar{\eta}) = m_1^2 + m_2^2\bar{\eta}\eta$ and m_1^2, m_2^2 are defined to be $\Delta(\Delta-d)$ and $(2\Delta-d)$ respectively and where $|g(x)|$ is the determinant of the metric on AdS.

Within this framework, the AdS/LCFT correspondence is written in the following form

$$\left\langle \exp\int d\bar{\eta}d\eta\int_{\partial\Omega} d^d\vec{x}\hat{O}(\vec{x},\eta,\bar{\eta})\Phi_b(\vec{x},\eta,\bar{\eta})\right\rangle = \exp\left(-S_{cl}[\Phi_b(\vec{x},\eta,\bar{\eta})]\right), \tag{7}$$

where $\Phi_b(\vec{x},\eta,\bar{\eta})$ is the value of $\Phi(x,\eta,\bar{\eta})$ on the boundary. Then the two point correlation functions were calculated [7], though the results were not in closed form and one had to expand $\Phi(x,\eta)$ in terms of $C(x)$, $\alpha(x)$, $\bar{\alpha}(x)$ and $D(x)$ to obtain correlation function.

To have non trivial 4 or higher point functions, one should add some kind of interaction to the action (6). In [8] various n-point correlations of superfield components were calculated at tree level by adding an interaction term to the free theory. Again, the results the correlation functions were not found in closed form. Also they had chosen a special form for interaction, more precisely, they have taken the interaction term to be proportional to $\prod_1^n \phi(x,\eta_i)$, but to keep the action BRST invariance, the sum of η_i should be zero. The choice made in [8] is to take half of η_i's equal to η and the other half equal to $-\eta$.[1] In this paper we will not restrict ourselves to a specific choice and will consider more general cases. It turns out that with the new action, nearly all the correlators will have correction, while in [8] only few term had correction. It is also important to see what happens when one goes beyond tree level. In ordinary CFT's, Witten diagrams have been introduced to calculate n-point functions, perturbatively. We will generalize this idea to LCFT's and will find proper Feynman rules to describe the diagrams.

In this paper we obtain n-point correlation functions of superfields in closed form instead of its components. This method leads us to generalization of Witten diagrams which deal with ordinary fields to the diagrams dealing with superfields. Also by this generalization we can evaluate loop corrections.

3 Interaction action and the generalized Witten diagrams

In order to find non trivial n-point functions at tree level and loop corrections, one should consider interaction terms in $\Phi(x,\eta,\bar{\eta})$ in addition to free theory. Furthermore we wish the action to have BRST symmetry so that the correlation functions remain invariant under BRST transformations. We consider the interaction term as

$$S^I[\Phi] = \int_\Omega d^{d+1}x\sqrt{|g(x)|}\int d\bar{\eta}_1 d\eta_1\cdots d\bar{\eta}_n d\eta_n \delta(\eta_1+\cdots+\eta_n)\delta(\bar{\eta}_1+\cdots+\bar{\eta}_n)$$

[1] Of course this can be done only when n is even, if n is odd, another choice should be made.

$$\left[\frac{1}{n!}\lambda(\eta_1,\cdots,\eta_n,\bar{\eta}_1,\cdots,\bar{\eta}_n)\Phi(x,\eta_1,\bar{\eta}_1)\cdots\Phi(x,\eta_n,\bar{\eta}_n)\right]. \quad (8)$$

In the language of superfield, the infinitesimal BRST transformation is of the form

$$\delta\Phi(x,\eta,\bar{\eta}) = (\bar{\epsilon}\eta + \bar{\eta}\epsilon)\Phi(x,\eta,\bar{\eta}), \quad (9)$$

where $\bar{\epsilon}$ and ϵ are infinitesimal anti-commuting parameters. Presence of Dirac delta functions in $S^I[\Phi]$ guarantees BRST symmetry of these actions, because

$$\delta\left[\Phi(x,\eta_1,\bar{\eta}_1)\cdots\Phi(x,\eta_n,\bar{\eta}_n)\right] = \left[\Phi(x,\eta_1,\bar{\eta}_1)\cdots\Phi(x,\eta_n,\bar{\eta}_n)\right]\left[\bar{\epsilon}\sum_{i=1}^{n}\eta_i + \sum_{i=1}^{n}\bar{\eta}_i\epsilon\right]. \quad (10)$$

With this interaction term, we will move on to establish suitable Fenyman rules for generalized Witten diagrams. To have all the rules, we should study three terms: first, the bulk-bulk Green function, second, the surface-bulk Green function and third the vertex related to the interaction term (8).

As we will see, it is better to rewrite the free action (equation (6)) in the following form, which is a more suitable form to derive Feynman rules,

$$S^f[\Phi] = -\frac{1}{2}\int_\Omega d^{d+1}x\sqrt{|g(x)|}\int d\bar{\eta}_1 d\eta_1\, d\bar{\eta}_2 d\eta_2\,\delta(\eta_1+\eta_2)\delta(\bar{\eta}_1+\bar{\eta}_2)$$
$$\left[\vec{\nabla}\Phi(x,\eta_1,\bar{\eta}_1).\vec{\nabla}\Phi(x,\eta_2,\bar{\eta}_2) + m^2(\eta_1,\eta_2,\bar{\eta}_1,\bar{\eta}_2)\Phi(x,\eta_1,\bar{\eta}_1)\Phi(x,\eta_2,\bar{\eta}_2)\right]. \quad (11)$$

From now on we drop $\bar{\eta}$ dependence of all entities except integrals measures for simplicity. Considering total action $S = S^f[\Phi] + S^I[\Phi]$, $\frac{\delta S}{\delta\Phi(x,\eta_1)}$=0 leads the equation of motion

$$\left(\nabla^2 - m^2(\eta_1)\right)\Phi(x,\eta_1) = -\int d\bar{\eta}_2 d\eta_2\cdots d\bar{\eta}_n d\eta_n\,\delta(\eta_1+\cdots+\eta_n)$$
$$\frac{1}{(n-1)!}\lambda(\eta_1,\cdots,\eta_n)\Phi(x,\eta_2)\cdots\Phi(x,\eta_n). \quad (12)$$

Introducing Dirichlet Green function for this system as

$$\left(\nabla^2 - m^2(\eta_1)\right)\mathcal{G}(x,y,\eta_1,\eta_2) = \frac{\delta(x-y)\delta(\eta_1+\eta_2)}{\sqrt{|g(x)|}}, \quad (13)$$

together with the boundary condition

$$\int d\bar{\eta}_2 d\eta_2\,\mathcal{G}(x,y,\eta_1,\eta_2)|_{x\in\partial\Omega} = 0, \quad (14)$$

and applying Green's theorem we obtain

$$\begin{aligned}\Phi(x,\eta_1) &= \int d\bar{\eta}_2 d\eta_2\int d^{d+1}y\sqrt{|g(y)|}\,\mathcal{G}(x,y,\eta_1,\eta_2)\left(\nabla^2 - m^2(\eta_2)\right)\Phi(y,\eta_2) \\ &+ \int d\bar{\eta}_2 d\eta_2\int_{\partial\Omega} d^d y\sqrt{|h(y)|}\,\Phi(y,\eta_2)n^\mu\frac{\partial}{\partial y^\mu}\mathcal{G}(x,y,\eta_1,\eta_2) \quad (15)\end{aligned}$$

where $|h(x)|$ is the determinant of the induced metric on $\partial\Omega$ and n^μ the unit vector normal to $\partial\Omega$ and pointing outwards. Note that because of the new form of free action, the Green function has both dependence on η_1 and η_2. This Green function is just the one derived in [7] multiplied by $\delta(\eta_1+\eta_2)=\eta_1+\eta_2$. That is

$$\mathcal{G}(x,y,\eta_1,\eta_2)=G(x,y,\eta_1)\delta(\eta_1+\eta_2), \tag{16}$$

where G is introduced in [7]. Note that as the dependence of this Green function on η is just through $\bar{\eta}\eta$, it doesn't matter to put η_1 or η_2 in the argument of G.

Now we will consider the surface-bulk Green function. Again, because there are two η's in the free action, this Green function will relate $\Phi(x,\eta_2)$, which lives in the bulk, to $\Phi_b(\vec{y},\eta_1)$, which lives on the boundary:

$$\Phi(x,\eta_1)=\int d\bar{\eta}_2 d\eta_2 \int d^d\vec{y}\,\mathcal{K}(x,\vec{y},\eta_1,\eta_2)\Phi_b(\vec{y},\eta_2). \tag{17}$$

Again, one can see that this Green function, is just the same as the one derived in [7], multiplied by $\delta(\eta_1+\eta_2)$, that is:

$$\mathcal{K}(x,\vec{y},\eta_1,\eta_2)=K(x,\vec{y},\eta_1)\delta(\eta_1+\eta_2)=a(\eta_1)\left(\frac{x^d}{(x^d)^2+|\vec{x}-\vec{y}|^2}\right)^{\Delta+\bar{\eta}_1\eta_1}\delta(\eta_1+\eta_2) \tag{18}$$

with

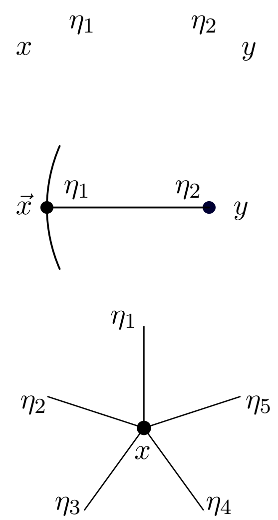
$$a(\eta_1)=\frac{\Gamma(\Delta+\bar{\eta}_1\eta_1)}{2\pi^{d/2}\Gamma(\alpha+1)}=a+a'\bar{\eta}_1\eta_1, \tag{19}$$

where $\alpha=\Delta+\bar{\eta}_1\eta_1-d/2$.

So far, we have derived Feynman rules for different propagators, the last step to make the discussion complete is derivation of the vertex term, which is present due to presence of interaction in the theory. As the interaction term is proportional to product of n Φ's, to each vertex n line's are attached, each of them carrying a different η. The η's are different because they are different in the interaction term. The only condition which should be satisfied, is that the sum of all η's of the vertex should vanish. This is done by putting proper delta functions on the vertex. The strength of the interaction is controlled by the coupling constant('s) $\lambda(\eta_i)$. Note that this is not a simple coupling constant, one should expand it in terms of different η's (and $\bar{\eta}$'s) to have the complete form.

We have briefly written the complete Feynman rules in figure 1. It worths to mention that to every point, a spacial coordinate is assigned, and to every ending of any propagator a grassmann variable is assigned. Like all Feynman rules, one should integrate over all undetermined spacial coordinates, in addition, one should integrate over all undetermined grassmann variables.

To see how powerful this method is, we will calculate two point function at tree level by considering following Feynman diagram (figure 2). This diagram is present in every interactive theory, in fact there is no interaction term present in the diagram. So the result should turn out to be the same as derived in free theory [2, 3, 7]. In [7] this was done using grassmann variables, but the final result was not in closed form.



a) $\Rightarrow$ $G(x,y,\eta_1)\delta(\eta_1+\eta_2)$

b) $\Rightarrow$ $\Phi_b(\vec{x},\eta_1)K(\vec{x},y,\eta_1)\delta(\eta_1+\eta_2)$

c) $\Rightarrow$ $\int d^{d+1}x \int d\bar{\eta}_i d\eta_i \sqrt{|g(x)|}\lambda(\eta_i)\delta(\sum \eta_i)$

Figure 1: Different Feynman rules of the theory: a) Bulk-bulk propagator, b) Boundary-bulk propagator, c) Interaction vertex (assuming the interaction term is proportional to Φ^5)

The term corresponding to the above Feynman diagram is read as

$$\begin{aligned}\Phi_b(\vec{y}_1,\eta_1)\Phi_b(\vec{y}_2,\eta_2)\int d^{d+1}x\sqrt{|g(x)|}\int d\bar{\eta}_1' d\eta_1' d\bar{\eta}_2' d\eta_2' \times \\ K(\vec{y}_1,x,\eta_1)K(\vec{y}_2,x,\eta_2)\delta(\eta_1+\eta_1')\delta(\eta_2+\eta_2')\delta(\eta_1'+\eta_2').\end{aligned} \tag{20}$$

Integrating over grassmannian variables leads to a single delta function on external η's, that is $\delta(\eta_1+\eta_2)$. This is a general result and guarantees BRST symmetry. We will discuss it more completely in the next section. There will remain a single integral over spatial coordinate. By Ads/CFT correspondence, the result should be equal to the two point function in the CFT part multiplied by the value of boundary fields: $\langle\hat{O}(\vec{y}_1,\eta_1)\hat{O}(\vec{y}_2,\eta_2)\rangle\Phi_b(\vec{y}_1,\eta_1)\Phi_b(\vec{y}_2,\eta_2)$.[2] The integration over x can be done [7] and the result is

$$\langle\hat{O}(\vec{y}_1,\eta_1)\hat{O}(\vec{y}_2,\eta_2)\rangle = \frac{a(\eta_1)}{|\vec{y}_1-\vec{y}_2|^{2\Delta+\bar{\eta}_1\eta_1}}\delta(\eta_1+\eta_2). \tag{21}$$

The result is consistent with the one derived before [2, 3, 7], in addition, we have found the correlation function in closed form. By expanding (21) in powers of η_i's, different correlation functions of the fields A, B, $\bar{\zeta}$ and ζ are obtained.

4 n-Point Correlation Functions at Tree Level

When there is some kind of interaction in the bulk theory, the n-point correlation function will find corrections. In this section, we will find the correction to the first order of λ or

[2] In fact, you have integrations over external coordinates and grassmanns on both side of equation, but one can drop them without loss of generality

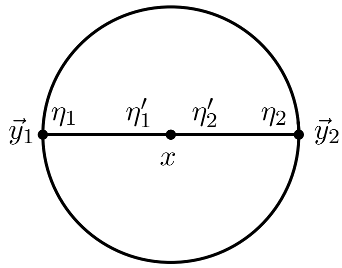


Figure 2: The tree level Feynman rule for two point function.

equivalently to tree level. Let's consider an n-point correlation function. In the framework of Witten diagrams, this means that we should have n distinct point on the boundary. Each line should connect to others to produce a connected diagram. The simplest way to do that, is to take an interaction action proportional to Φ^n. In this case, one can connect all the points on the boundary to an n-vertex in the bulk (see figure 3 for the case n=4).

This diagram is equivalent to a term proportional to Φ_b^n. On the other hand, by expanding exponential in the left hand of equation (7), there will be a term having the same factor in the following form:

$$\frac{1}{n!}\int d\vec{y}_1\cdots d\vec{y}_n \int d\bar{\eta}_1' d\eta_1' \cdots d\bar{\eta}_n' d\eta_n' \left\langle \hat{O}(\vec{y}_1,\eta_1')\cdots\hat{O}(\vec{y}_n,\eta_n')\right\rangle \\ \Phi_b(\vec{y}_1,\eta_1')\cdots\Phi_b(\vec{y}_n,\eta_n'). \tag{22}$$

So, using the Feynman rules, the correction to n-point correlation function of the operators

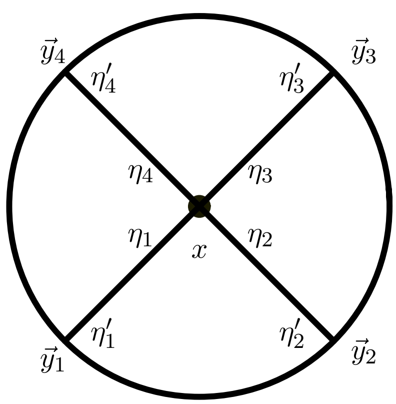


Figure 3: The tree level correction to correlation function in the case $n = 4$

$\hat{O}(\vec{y},\eta')$ is read to be

$$\left\langle \hat{O}(\vec{y}_1,\eta'_1)\cdots\hat{O}(\vec{y}_n,\eta'_n)\right\rangle = \int d^{d+1}x\sqrt{|g(x)|}\int d\bar{\eta}_1 d\eta_1\cdots d\bar{\eta}_n d\eta_n \delta(\eta_1+\cdots+\eta_n)$$
$$\lambda(\eta_1,\cdots,\eta_n)K(x,\vec{y}_1,\eta_1)\cdots K(x,\vec{y}_n,\eta_n)\delta(\eta_1+\eta'_1)\cdots\delta(\eta_n+\eta'_n) \qquad (23)$$

The integrations on grassmann variables can be done easily, the result is a single delta function on the external η's. After substituting $K(x,\vec{y}_i,\eta_i)$ from Eq. (18), we obtain n-point correlation function at tree level

$$\left\langle \hat{O}(\vec{y}_1,\eta'_1)\cdots\hat{O}(\vec{y}_n,\eta'_n)\right\rangle = a(\eta'_1)\cdots a(\eta'_n)\int d^{d+1}x\sqrt{|g(x)|}\delta(\eta'_1+\cdots+\eta'_n)$$
$$\lambda(\eta'_1,\cdots,\eta'_n)\left(\frac{x^d}{(x^d)^2+|\vec{x}-\vec{y}_1|^2}\right)^{\Delta+\bar{\eta}'_1\eta'_1}\cdots\left(\frac{x^d}{(x^d)^2+|\vec{x}-\vec{y}_n|^2}\right)^{\Delta+\bar{\eta}'_n\eta'_n}. \qquad (24)$$

We would like to have some comment on this correction. First of all, it is easy to check that the result is conformal covariant. You should just note that the functions K transform properly under scaling, just you should note that it is possible to rescale the dummy variable x. Second, note that there exists a delta function on sum of external grassmann variables. As said before, it guarantees BRST symmetry. Let's be more precise, consider the n-point correlation function $X = \left\langle \hat{O}(\vec{y}_1,\eta_1)\cdots\hat{O}(\vec{y}_n,\eta_n)\right\rangle$. Under BRST transformation, $\Phi(\eta)\to(1+\bar{\epsilon}\eta)\Phi(\eta)$, we have

$$\delta X = \bar{\epsilon}(\eta_1+\cdots+\eta_n)X. \qquad (25)$$

Due to BRST symmetry, the above expression should vanish. So, if the sum of η's is non zero, the correlator should vanish. Consequently, the correlator should have a delta function on η's as a multiplier.

The other point to be clarified is that our coupling constant is a compound one. We should expand it in terms of η's (and $\bar{\eta}$'s) to find the components. As an example we consider the Φ^3 theory. The coupling constant is written as

$$\lambda(\eta_1,\eta_2,\eta_3,\bar{\eta}_1,\bar{\eta}_2,\bar{\eta}_3) = \lambda_0+\bar{\eta}_i\eta_j\lambda_{ij}+\bar{\eta}_i\eta_j\bar{\eta}_k\eta_l\lambda_{ijkl}+\bar{\eta}_i\eta_j\bar{\eta}_k\eta_l\bar{\eta}_m\eta_n\lambda_{ijklmn}. \qquad (26)$$

There is no other term in this series because our variables are grassmanns. Further investigation shows that the last term is irrelevant. This happens because we have a delta function on η's which imposes a constraint on the garassmans. So one of the grassmann can be written in terms of the others and the last term in equation (26) vanishes. Also in the expansion of λ we have not written terms like $\bar{\beta}_i\eta_i$ as there should not be any grassmannian constant in the theory.

With this coupling constant('s) we can find out what vertices are present. As the last term is absent, we do not have the CCC vertex. This leads to vanishing correlator of $\langle\hat{A}\hat{A}\hat{A}\rangle$ in CFT part, which is a desired result. Also vertices which have odd number of grassmann fields do not exist, but other vertices are present in the theory.

Let's investigate the Φ^3 theory more precisely and derive the three point functions explicitly. This is a good check for our theory, as we know the form of correlators from

symmetry. Also, knowing the three point functions, one can say something about OPE's of the fields present in the theory.

In this case the integrations can be done explicitly

$$
\begin{aligned}
&\langle \hat{O}(\vec{y}_1,\eta_1)\hat{O}(\vec{y}_2,\eta_2)\hat{O}(\vec{y}_3,\eta_3)\rangle = \\
&a(\eta_1)a(\eta_2)a(\eta_3)\delta(\eta_1+\eta_2+\eta_3)\lambda(\eta_1,\eta_2,\eta_3)\frac{\pi^{\frac{d}{2}}}{2}\frac{\Gamma\left(\frac{3\Delta}{2}-\frac{d}{2}+\frac{\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2+\bar{\eta}_3\eta_3}{2}\right)}{\Gamma(\Delta+\bar{\eta}_1\eta_1)\Gamma(\Delta+\bar{\eta}_2\eta_2)\Gamma(\Delta+\bar{\eta}_3\eta_3)} \\
&\frac{\Gamma\left(\frac{\Delta}{2}+\frac{-\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2+\bar{\eta}_3\eta_3}{2}\right)\Gamma\left(\frac{\Delta}{2}+\frac{\bar{\eta}_1\eta_1-\bar{\eta}_2\eta_2+\bar{\eta}_3\eta_3}{2}\right)\Gamma\left(\frac{\Delta}{2}+\frac{\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2-\bar{\eta}_3\eta_3}{2}\right)}{y_{12}^{\Delta+\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2-\bar{\eta}_3\eta_3}\,y_{13}^{\Delta+\bar{\eta}_1\eta_1-\bar{\eta}_2\eta_2+\bar{\eta}_3\eta_3}\,y_{23}^{\Delta-\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2+\bar{\eta}_3\eta_3}},
\end{aligned} \tag{27}
$$

where $y_{ij}=|\vec{y}_i-\vec{y}_j|$. Expanding both sides of the above equation, different correlators are obtained. The expressions are very huge and we will bring here only some of them

$$
\begin{aligned}
\langle \hat{A}(\vec{y}_1)\hat{A}(\vec{y}_2)\hat{B}(\vec{y}_3)\rangle &= -\langle \hat{A}(\vec{y}_1)\hat{\bar{\zeta}}(\vec{y}_2)\hat{\zeta}(\vec{y}_3)\rangle = \lambda_0 b_1\frac{1}{y_{12}^{\Delta}y_{13}^{\Delta}y_{23}^{\Delta}} \\
\langle \hat{\zeta}(\vec{y}_1)\hat{\bar{\zeta}}(\vec{y}_2)\hat{B}(\vec{y}_3)\rangle &= \left(\frac{a'}{a}\lambda_0+\lambda_{12}+\lambda_{13}+\lambda_{32}+\lambda_{33}\right)b_1\frac{1}{y_{12}^{\Delta}y_{13}^{\Delta}y_{23}^{\Delta}} \\
&\quad +\lambda_0\frac{1}{y_{12}^{\Delta}y_{13}^{\Delta}y_{23}^{\Delta}}\left(b_2+b_3\ln\frac{y_{13}y_{23}}{y_{12}}\right)
\end{aligned} \tag{28}
$$

where b_1, b_2 and b_3 are some constants depending on Δ, and a and a' are defined in equation (19). Other correlation functions have similar structure and are consistent with previous results [5, 6].

5 Loops

In this last section, we briefly investigate the general properties of arbitrary generalized Weitten diagrams which may contain loops. Though nearly none of them can be calculated explicitly, still there is a lot to say. As an example consider the Φ^4 and the one loop correction to four point function. The corresponding diagram is shown in figure 4. The term associated to this diagram can be read easily: we have four lines joining bulk to boundary, two lines in the bulk and two vertices. The expression is too lengthy, so we will not bring it here.

Most of integrations on η_i's and η_j''s can be done easily due to presence of delta functions. Just like before, the over all result has a delta function on sum of external grassmann variables, which means BRST invariance. This leads to vanishing correlators which have only $\hat{A}(\vec{y})$. To see how this happens, it is sufficient to note that the delta functions on grassmann variables can be written in the following form

$$\delta(\eta_1+\eta_2)=\eta_1+\eta_2. \tag{29}$$

So, in front of the correlator, there stands a factor like $\sum\eta_i$ and there is no term in the correlator that has no dependence on η_i. This is just the term consist of $\hat{A}$'s, only. This result has been seen before [16, 5], and has origin in the fact that the expectation value of the unity operator vanishes.

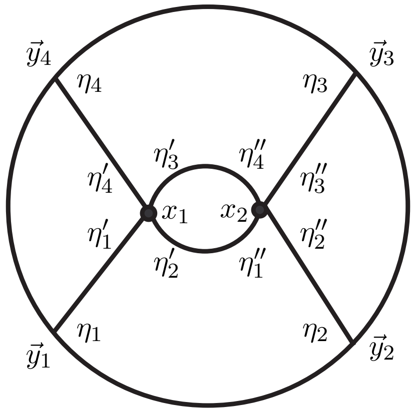


Figure 4: The one-loop correction to four point function in the Φ^4 theory.

There will remain three integrations, two over spatial coordinates x_1 and x_2, and the other over one grassmann variable (and its complex conjugate) which play a role like the circulating momenta in the usual Feynman diagrams.

To check conformal invariance, one should know how the functions K and G change under these transformations. Transformation of K is easily derived from it's form (equation (18)): it rescales and its weight is $\Delta + \bar{\eta}\eta$. To see this in the case of simple scaling, it is enough to rescale the dummy variables x_1 and x_2. Under special conformal transformation, with similar but more complicated calculation, one can show the same result is obtained [17]. The bulk-bulk propagators, G, happen to be invariant under scalings. This is shown by [17] in ordinary CFT, generalization to LCFT is straightforward, using grassmann variables. This shows that the diagram sketched above, has conformal invariance. It is clear that this happens to all diagrams of any order and any number of loops and conformal symmetry is preserved in this loop expansion.

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